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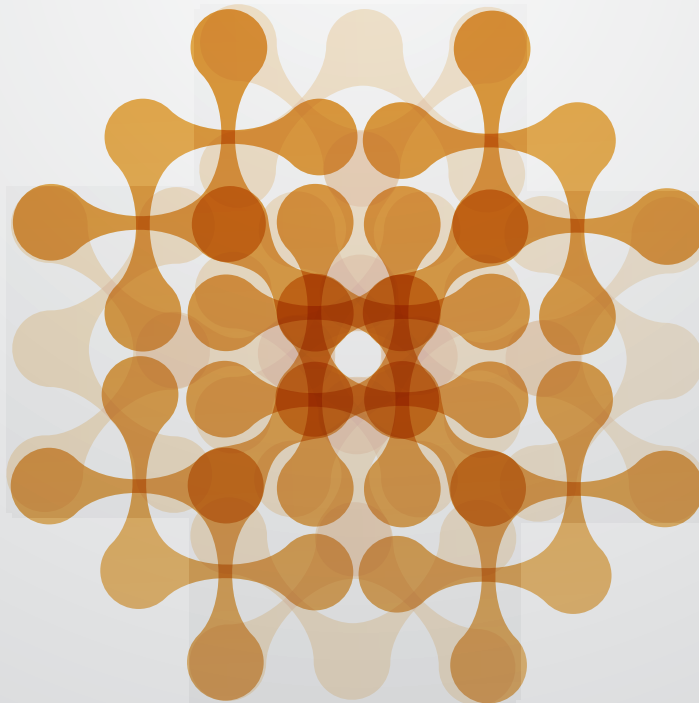
Credit, Misallocation and TFP: Mexico in the 21st Century[*]

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Abstract

We study the relation between misallocation of resources, TFP and credit conditions in Mexican manufacturing at the industry level. We develop a tractable theoretical framework to account for TFP changes in the Mexican manufacturing sector due to changes in distortions in the use of capital, labor and intermediate goods. We show that these distortions account for a large fraction of aggregate TFP changes in the period. Reduced form estimations reveal that changes in distortions in the data are driven by changes in the availability and the cost of credit. We build a general equilibrium model that maps financial frictions into distortions. The model accounts for a large fraction of observed TFP growth between 2003-2010. The contribution of financial factors varies from over 90% of model predicted TFP growth in the expansionary years of 2005-2008 to about 15% of the downturn in 2009. The recovery of 2010 was largely fuelled by a reduction in sectoral financial frictions, particularly a fall in interest rates and the resulting reallocation of credit to less distorted sectors.

1 Introduction

The relationship between output growth and total factor productivity is well established. The earliest calculations of Solow (1957) attributed only 12% of the growth in the United States between 1900 and 1949 to the accumulation of factors of production, and the remaining 88% to the "residual". Subsequent work enlarging the scope and time period of these estimates reduced the size of the residual to about 50% (see for example Klenow and Rodriguez Clare 1997), however the primacy of the contribution of TFP to growth is undeniable. Movements in TFP have played an important role in recent growth miracles in India (Bollard et. al 2013) and China (Dekle and Vandenbroucke 2010). The reverse has also been true, large drops in output during "sudden stops" have been accompanied by a large fall in TFP (Calvo 2006).

Some recent studies have highlighted the role of input misallocation as an important factor behind these aggregate TFP changes. Pratap and Urrutia (2012) show that financial frictions can propagate interest rate shocks into measured TFP fluctuations by distorting the use of inputs in the economy. Benjamin and Meza (2009) analyze the real effects of Korea's 1997 crisis and find that a reallocation of resources towards low productivity sectors at this time generates a fall in TFP.

A recent strand of literature seeks to explain the differences in TFP levels between countries by emphasizing the role of firm-specific distortions, i.e. implicit taxes, barriers and constraints which result in the suboptimal allocation of resources and lower TFP levels (see, for instance, Restuccia and Rogerson 2008, Hsieh and Klenow 2009, Bartelsman et. al 2013). Sandleris and Wright (2014) use these same insights to understand changes in TFP over time, using firm level data from Argentina in the period around the 2001 crisis. Chen and Irarrazabal (2013) perform a similar analysis for the Chilean manufacturing sector after the 1982 debt crisis. These papers however do not investigate the reasons behind these distortions, as their focus is on quantifying their effects.

The goal of this paper is to understand the role of credit and financial frictions in accounting for the misallocation of resources and the changes in TFP over time. As in Pratap and Urrutia (2012), our focus is on financial frictions distorting firms' decisions to purchase inputs. However, as in the literature reviewed in the previous paragraph, we analyze the relation between credit and distortions at the micro level, instead of the aggregate macro level. In this sense, our work relates to Buera and Moll (2012), who extend the business cycle accounting methodology to economies with heterogeneous firms and show that credit shocks and financial frictions can be mapped into measured TFP and other aggregate wedges or distortions.[1]

As an empirical contribution, we construct a novel data set by linking manufacturing activity in Mexico with credit flows at a disaggregated level. Our data encompasses 82 sectors of activity for the period from 2003 to 2010. This is a particularly interesting time frame to study, since it includes both the period of rapid growth of 2003-08, the economic crisis of 2008-09 and the subsequent recovery. We use this data to construct two measures of input distortions: (i) a distortion to the use of intermediate goods (ii) a distortion to the capital to output ratio. We construct a simple framework to show how these two distortions affect aggregate TFP.

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[1] Chari, Kehoe and McGrattan (2007) introduced the methodology of business cycle accounting as a way of inferring the aggregate levels of distortions that an economy faces and how they evolve over time. The idea is to infer these distortions from the wedges obtained from the optimality conditions of a standard neoclassical model. For example, the labor wedge is defined as the ratio between the marginal rate of substitution of consumption for leisure and the marginal product of labor, which should be equal in an undistorted economy.

Reduced form analysis shows that these distortions are linked to differences in credit availability and interest rates across sectors and over time. We therefore build a general equilibrium model that maps financial frictions into distortions to see how changes in financial frictions affect aggregate TFP. Firms face a working capital constraint and a borrowing constraint. The former implies that firms have to finance their input purchases using bank credit or the more expensive trade credit while the latter constraint dictates the amount of bank credit available to firms. Taken together, these constraints dictate that the availability and costs of credit affect firm purchases of inputs. We feed the model a sequence of interest rate shocks, financial availability shocks (as indicated by the tightness of the borrowing constraint) and productivity shocks, all calibrated form the real and financial data at the sectoral level.

Our TFP decomposition suggests that deviations from the optimal use of intermediate goods and from the optimal capital labor ratio account for a substantial amount of TFP variation in this period. These distortions are strongly linked to variations in credit and interest rates across sectors. Our model which incorporates these variations in financial frictions can account, in large part for the observed movements in TFP in the period. In particular, we can account for over 90% of the TFP growth in 2003-2008. The model slightly overpredicts the fall in TFP in 2009 and the subsequent recovery of 2010. Financial factors and the reallocation of credit across sectors play a large in role explaining changes in TFP. The contribution of financial factors ranges from a modest 20% of TFP variation in 2003-2005, to about 96% of the growth in TFP in the expansionary years of 2005-2008. Interest rates and credit availability did not play a large role in explaining the downturn of 2008-2009 (about 15% of TFP growth), but the subsequent recovery was largely fuelled by a fall in interest rates and more importantly, an improved allocation of credit across sectors. A common factor explaining TFP growth throughout the period is the reallocation of credit in a manner that reduces distortions and improves productivity.

The paper is organized as follows. In Section 2 we describe the data used to analyze the relationship between economic activity and credit and present aggregate statistics. Section 3 sets out an analytical framework to account for aggregate TFP changes through changes in technology, reallocation of labor and changes in sectoral distortions. We apply this methodology to our dataset for Mexican manufacturing and find that changes in distortions play a large role in TFP fluctuations. Section 4 presents a simple model with financial frictions linking credit variables and sectoral distortions and the results of panel regressions suggesting a robust relation between these variables in our dataset. Finally, in Section 5 we calibrate and simulate the full equilibrium model subject to shocks to credit conditions taken from the data and discuss its implications for aggregate TFP.

2   Data and Stylized Facts

A major contribution of our paper is the construction of a data set that links manufacturing activity with bank lending to firms. In this section we describe our two main data sources and our procedure to merge them. We also describe some stylized facts for the manufacturing sector during the period 2003-10 obtained from our combined database. In particular, we show a large increase in total factor productivity (TFP) from 2004-2008, a contraction in 2009 related to the world financial crisis, and a small recovery in 2010. At the same time real short term credit to manufacturing increased during the rapid growth period, especially from 2005 to 2008, and collapsed in 2009 and 2010, with no observed recovery at the end of the period.

2.1 Dataset Construction

We have two main data sources: The first is the annual industrial survey (EIA for its acronym in Spanish) collected by the Mexican statistical agency INEGI. The second source is the loan portfolio of all commercial banks, known as the R04C, maintained by the banking regulatory authority, the *Comision Nacional Bancaria y de Valores* (CNBV). Confidentiality restrictions prohibit us from analyzing the data at the establishment or loan level. We therefore work at the lowest level of aggregation currently feasible, namely at the 4-digit industry level, following the 2007 North American Industrial Classification System (NAICS). The banking data are at a monthly frequency, whereas the establishment level data are collected yearly. We therefore aggregate the loan data to the annual level. A further complication is that while the EIA uses the NAICS throughout the sample, the R04C data is classified according to an internal classification system for the period up to July 2009 and to the NAICS thereafter. We construct a flexible, probabilistic crosswalk between these two systems, details of which are given in Appendix A.1.

The EIA data is a representative sample of nearly 7000 manufacturing establishments. We were able to get data on 86 sectors at the 4-digit level and 231 subsectors at the 6-digit level.[2] We use data on gross output and expenditure on intermediate goods to construct measures of value added. The labor input is measured as the number of people hired directly and indirectly by the establishment. The capital input is constructed using the perpetual inventory method, using information on investment. Appendix A.2 provides a more detailed description of the variables used from EIA.

The R04C dataset is the universe of loans by commercial banks to firms. This data is outstanding loan balances, collected at the loan level, and is available at a monthly frequency. We construct a measure of credit flow by looking at the debt outstanding on all new loans (i.e. loans with dates of disbursement in the month in which the data is collected) in a particular sector in a particular month. This gives us information on how much credit was disbursed to each sector in each period. Our measure of credit includes outstanding interest payments.[3] We focus on data for short term credit, defined as credit which matures in a period of 12 months or less.[4] This accounts for, on average, 85% of all bank credit. Finally, we construct cost of credit measures by looking at average real interest rates paid by sector, weighted by the size of the loan in total credit flow in the corresponding period and deflated by the change in the manufacturing price level.

All nominal variables, with the exception of intermediate goods are deflated by the producers price index for manufacturing published by the INEGI. Intermediate goods are deflated by an intermediate goods index.

2.2 Aggregate Stylized Facts: Output and TFP

We calculate aggregates by adding up industry level variables. We focus on 2003-2010, the period in which our two databases overlap. As mentioned earlier, this period allows us to study two phenomena: high growth in output and TFP in the first five years and a contraction in the last two periods as the international financial crisis affected the Mexican economy.

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[2] Aggregates of subsectors containing 4 or fewer firms were not provided to us in the interests of confidentiality.

[3] We also constructed a measure of credit that excludes interest payments which behaves very similarly to the other measure at the aggregate and sectoral level.

[4] As we will see in the next sections, we consider financial frictions which affect the amount of working capital to which firms have access. The data counterpart of that is short term credit.
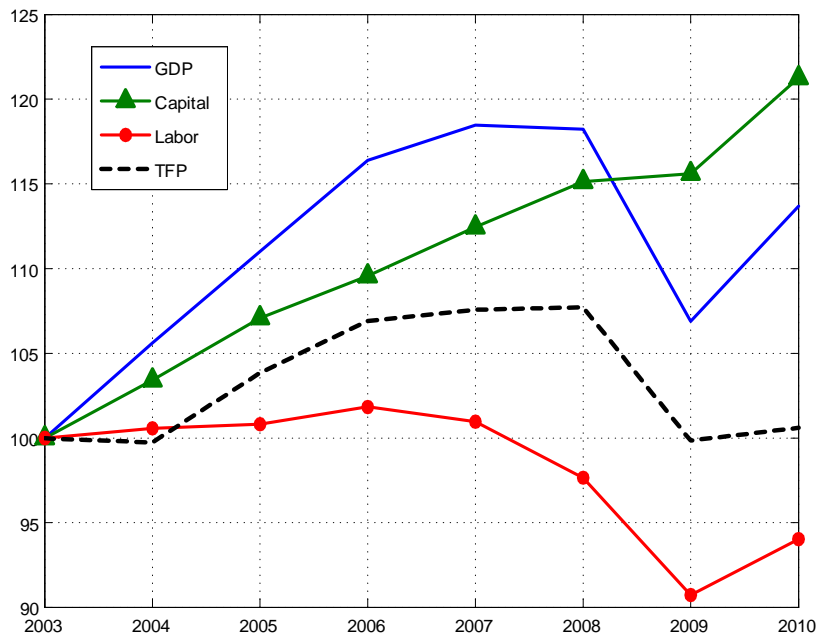
Figure 1: Real Output, Inputs and Total Factor Productivity (2003=100)

Figure 1 shows real manufacturing GDP, capital and labor inputs from our sample, relative to the year 2003. The former is calculated by adding up real value added in all sectors, whereas the inputs are the stock of capital and personnel employed aggregated over sectors. The figure illustrates the manufacturing boom between 2003 and 2008, when output grew at an annual average of 2.5%. Interestingly labor input was relatively stagnant in this period, suggesting that the sources of growth lay in productivity.[5] This period of expansion came to an abrupt halt in 2009, when output contracted by almost 11%. This fall in output was accompanied by a smaller drop in labor of about 7%. Investment slowed down substantially between 2008 and 2009, leading to a decceleration in the growth of capital stock. The recovery in the following year was modest, as output and labor inputs grew by less than 4%.

Figure 1 also shows the evolution of aggregate total factor productivity (TFP) over the period, relative to 2008. Assuming a Cobb Douglas production function, aggregate TFP is defined as

$$A = \frac{Y}{K^\alpha L^{1-\alpha}}$$

where Y represents real value added (manufacturing GDP) and K and L measure aggregate inputs of capital and labor respectively. The aggregate labor share $1 - \alpha$ is constructed as a weighted average of the labor share of income in each sector.[6] The weights are the share of each sector in total output. Figure 1 shows that TFP increased between 2004 and 2008, mirroring the increase in output. TFP fell in 2009 by 7.3%

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[5] We verified this fact by looking at employment in manufacturing in a completely different database, the National Survey of Occupation and Employment (ENOE for its acronym in Spanish). The employment series from EIA and from ENOE exhibit very similar behavior.

[6] For reasons elaborated in Section 3, these income shares are taken from the corresponding sectors in the U.S.
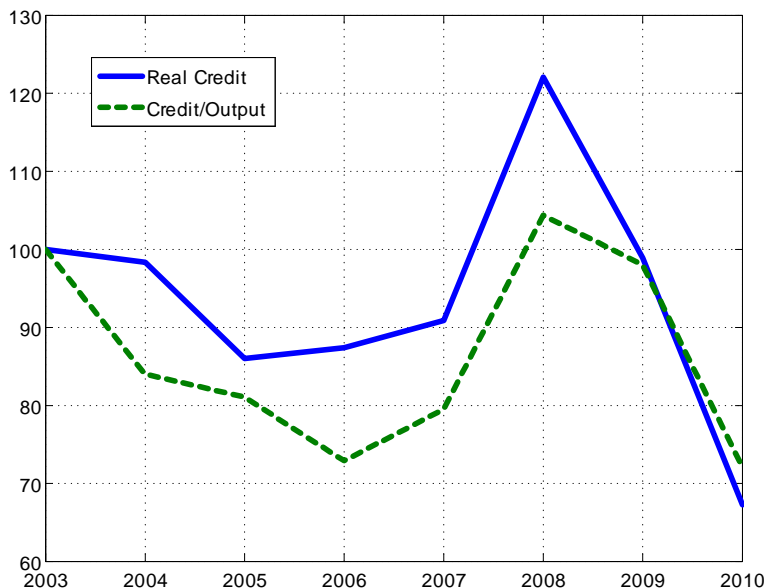
Figure 2: Real Credit Flow and Credit Intensity (2003=100)

and recovered slightly in the following year. This figure shows the importance of the role of total factor productivity in output fluctuations.

2.3   Aggregate Stylized Facts: Credit Flows and Interest Rates

Figure 2 shows the measure of short term credit flow described earlier. The aggregate flow of credit, which was declining in the first few years of the sample, increased between 2005 and 2008, and dramatically in the 2007-2008 period.[7] The fall in output in 2009 was also reflected in a fall in aggregate credit. Interestingly, the recovery in aggregate output was not accompanied by a recovery in credit.

One natural question is whether these changes in credit flows reflect supply or demand factors. Probably both. Still, some evidence suggests that the supply of credit by banks has played a leading role. We look at the behavior of *credit intensity*, measured as the ratio of credit flows to gross output. As also shown in Figure 2, the expansion in real credit is accompanied by an increase in credit intensity, and the credit contraction in 2009-10 also shows as a fall in this ratio. Therefore, credit did not just respond to the cyclical behavior of gross output.

We additionally provide information on the behavior of weighted-average, real interest rates. Figure 3 reports the average across sectors, for each year. From a high of 4% in 2004 the real interest rate fell

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[7]At this point it is important to note that, since the financial crisis of 1994, Mexico has a low degree of financial intermediation. The private credit to GDP ratio was 14% in 2003, at the beginning of our period of analysis (see Kehoe and Meza 2011). This number is much lower than in similar Latin American economies (61% in Chile, for instance). It is possible that, starting from the historical low levels of credit in Mexico, the expansion of credit might reduce misallocation, as firms had more access to credit and were possibly able to come closer to efficient levels of factor use.
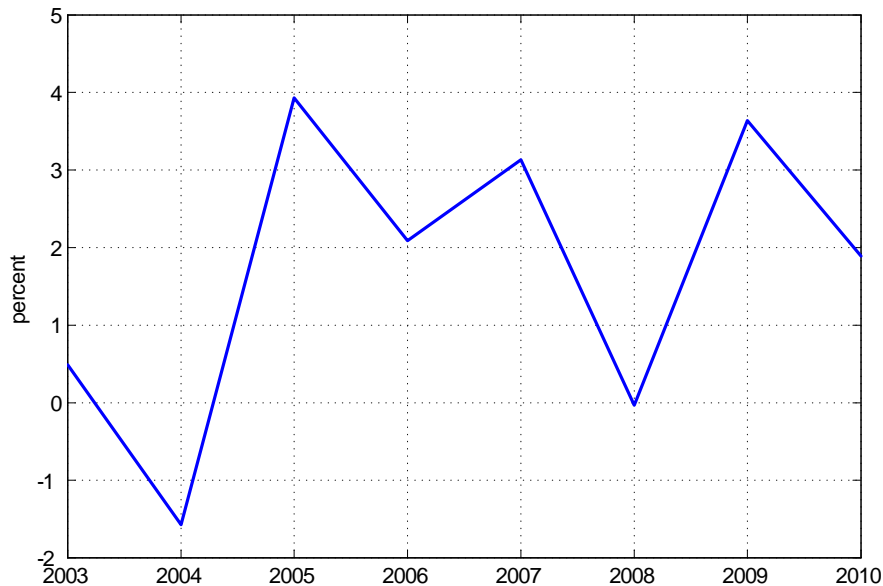
Figure 3: Average Real Interest Rate

till 2008 when it was close to zero. The international financial crisis was reflected in a sharp increase in interest rates to 3%, which fell by a percentage point in the next year of subsequent recovery. Notice that the negative correlation between the aggregate credit to output ratio and the average interest rate suggests again an important role for supply factors driving credit availability in this period.

The aggregate picture, while informative, obscures sectoral allocation issues. In the following section we set up a simple framework to measure distortions at the sectoral level from optimal input use and show how they relate to aggregate TFP.

3   Sectoral Distortions and Total Factor Productivity

We consider a simple, partial equilibrium model of multi-sector production with intermediate goods and sector specific distortions. These distortions introduce wedges between factor prices and marginal products. We consider: (i) a distortion affecting the capital to labor ratio; and (ii) a distortion affecting the ratio of intermediate goods to output. In this framework, we obtain an expression relating changes in aggregate TFP to the level of distortions and their growth rate. We perform an accounting exercise using our dataset and find a strong role for sectoral distortions in explaining changes in TFP over time for Mexican manufacturing.

3.1   Production and Firm's Optimization

Manufacturing activity, which we refer to as the "aggregate" in what follows, is composed of a discrete number n of sectors operating under perfect competition. Each sector is characterized by a representative firm operating under constant returns to scale, which produces a differentiated good. Firms rent capital

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K and labor L and combine them with intermediate goods M to produce (gross) output, according to the production function

$$Y_t^i = A_t^i \left[\left(K_t^i \right)^{\alpha^i} \left(L_t^i \right)^{1-\alpha^i} \right]^{\varepsilon^i} \left(M_t^i \right)^{1-\varepsilon^i} \qquad i = 1, ..., n. \tag{1}$$

Parameter A^i is a sector specific technology parameter, possibly changing over time. Factor shares α^i and ε^i are assumed to be constant over time, but are also sector specific.

Each sector faces two static, sector-specific distortions: (i) a distortion affecting the ratio of intermediate goods to output, that we model as a tax on intermediate goods; and (ii) a distortion affecting the capital to labor ratio, that we model as a tax on capital.[8] In each period, the profits of sector i's representative firm problem are defined as

$$\pi^i \equiv p^i Y^i - \left(1 + \tau_K^i \right) r K^i - w L^i - \left(1 + \tau_M^i \right) p^M M^i$$

omitting the time subscript from now on. Maximizing profits subject to the production function constraint gives us the following first order conditions:

$$\theta_{K,L}^i \equiv 1 + \tau_K^i = \frac{\alpha^i}{1 - \alpha^i} \left(\frac{w}{r} \right) \frac{L^i}{K^i} \qquad \theta_{M,Y}^i \equiv 1 + \tau_M^i = \left(1 - \varepsilon^i \right) \left(\frac{p^i}{p^M} \right) \frac{Y^i}{M^i}$$

In a world without financial frictions these two distortions disappear ($\theta_{K,L}^i = \theta_{M,Y}^i = 1$) and the standard first order conditions equating factor prices to marginal products would be recovered. For now, we will take these distortions as exogenous, as well as the sequences for factor prices (w_t, r_t, p_t^M) and sectoral output prices (p_t^i).

3.2 Aggregating Sectors

We define aggregate output (at constant prices) as the sum across sectors of the value of output:

$$Y \equiv \sum_{i=1}^n p_0^i Y^i$$

and, similarly, aggregate inputs as

$$K \equiv \sum_{i=1}^n K^i, \qquad L \equiv \sum_{i=1}^n L^i, \qquad M \equiv \sum_{i=1}^n M^i.$$

We also define (value-added) aggregate total factor productivity as

$$TFP \equiv \frac{Y - p_0^M M}{K^\alpha L^{1-\alpha}}$$

using as the aggregate share of capital an average of the sectoral shares weighted by value added

$$\alpha = \sum_{i=1}^n \omega^i \alpha^i, \qquad \omega^i \equiv \frac{p_0^i Y_0^i - p_0^M M_0^i}{Y_0 - p_0^M M_0}.$$

Assuming that factor shares *and* relative prices remain constant over time, we can write aggregate TFP in growth rates,

$$\widetilde{TFP} = \left(\frac{Y_0}{Y_0 - p_0^M M_0} \right) \widetilde{Y} - \left(\frac{p_0^M M_0}{Y_0 - p_0^M M_0} \right) \widetilde{M} - \alpha \widetilde{K} - (1 - \alpha) \widetilde{L} \tag{2}$$

where $\widetilde{x}_t = \log \left(\frac{x_t}{x_0} \right) \approx \frac{x_t - x_0}{x_0}$. The assumption of constant relative prices will be relaxed in Section 5. For now, it just allow us to focus on the role of sectoral distortions and reallocation of factors across sectors.

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[8] The model would allow us to identify a third distortion, for example one affecting the ratio of employment to output. However, it is easy to show that only changes in the two distortions that we are considering affect the growth rate of aggregate total factor productivity once we take into account reallocation effects.

3.3 Decomposing Aggregate TFP changes

In Appendix A.3 we show how we can combine firms' optimization first order conditions with the previous equation (2) to obtain

$$\widetilde{TFP} = \sum_{i=1}^{n} \left\{ \omega^i \left(\frac{1}{\varepsilon^i} \right) \left(\widetilde{A^i} \right) + \Phi_L^i \widetilde{L^i} + \Phi_{K,L}^i \widetilde{\theta_{K,L}^i} + \Phi_{M,Y}^i \widetilde{\theta_{M,Y}^i} \right\} \tag{3}$$

where ω^i is the share of the sector in aggregate value added, and

$$\Phi_L^i \equiv \omega^i - \alpha \left(\frac{K_0^i}{K_0} \right) - (1 - \alpha) \left(\frac{L_0^i}{L_0} \right),$$

$$\Phi_{K,L}^i \equiv \alpha \left(\frac{K_0^i}{K_0} \right) - \omega^i \alpha^i, \qquad \Phi_{M,Y}^i \equiv \frac{p_0^M M_0^i}{Y_0 - p_0^M M_0} - \omega^i \left(\frac{1 - \varepsilon^i}{\varepsilon^i} \right).$$

Equation (3) provides a useful expression decomposing aggregate TFP changes in four components:

1. Changes in sectoral technologies: $\sum_{i=1}^{n} \omega^i \left(\frac{1}{\varepsilon^i} \right) \widetilde{A^i}$.

2. Reallocation of labor across sectors: $\sum_{i=1}^{n} \Phi_L^i \widetilde{L^i}$.

3. Changes in sectoral distortions to the capital to labor ratio: $\sum_{i=1}^{n} \Phi_{K,L}^i \widetilde{\theta_{K,L}^i}$.

4. Changes in sectoral distortions to the intermediates to output ratio: $\sum_{i=1}^{n} \Phi_{M,Y}^i \widetilde{\theta_{M,Y}^i}$.

To gain some intuition, notice first that in an undistorted economy the three coefficients (Φ_L^i, $\Phi_{K,L}^i$, $\Phi_{M,Y}^i$) are zero for all sectors and aggregate TFP is driven only by changes in sectoral technologies.[9] We can also show using the definition of the aggregate labor share α that $\sum_{i=1}^{n} \Phi_L^i = 0$. This implies that changes in average employment in the same proportion for all sectors do not affect TFP, only labor reallocation across sectors does. A sector which increases its relative employment size would contribute to increase aggregate TFP if and only if $\Phi_L^i > 0$, i.e., if and only if

$$\omega^i > \alpha \left(\frac{K_0^i}{K_0} \right) + (1 - \alpha) \left(\frac{L_0^i}{L_0} \right)$$

The condition implies that a sector that increases its employment contributes positively to TFP if its share of aggregate value is larger than its input shares, relative to the other sectors. This could occur because the sector is highly productive (i.e. it has a higher value of A^i) or purchases a suboptimal level of inputs. In other words, as expected, reallocation of labor towards more productive and less distorted sectors increases TFP.

Similarly, we can show that $\sum_{i=1}^{n} \Phi_{K,L}^i = 0$. Therefore, a change in the average level of the capital distortion which affects all sector equally does not impact aggregate TFP. Only changes in distortions which are heterogeneous across sectors do because of their impact in the dispersion of capital to labor ratios. A sector which increases its capital distortion $\theta_{K,L}^i$ (and therefore decreases its capital to labor ratio) would reduce aggregate TFP as long as $\Phi_{K,L}^i < 0$, i.e., as long as

$$\frac{K_0^i}{K_0} < \frac{\omega^i \alpha^i}{\alpha}$$

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[9] We will not have anything interesting to say about the technology component, that we take as exogenous. In a more disaggregated framework this component could be also a function of the distribution of *firm-level* distortions within a sector, which we are omitting in our analysis.

or that its initial capital stock is smaller than its relative returns to capital parameter α^i would warrant. This means that an increase in the distortion to the capital labor ratio will depress TFP for sectors which were already distorted initially.

Movements in the distortions to the use of intermediate goods affects aggregate TFP differently. The reason is that we are considering value-added TFP, as opposed to gross output TFP, so in general $\sum_{i=1}^{n} \Phi_{M,Y}^i \neq 0$. In our setup, an increase in the intermediates distortion $\theta_{M,Y}$ reduces aggregate TFP if and only if $\Phi_{M,Y}^i < 0$, i.e., if and only if $\theta_{M,Y}^i > 1$ so that the initial distortion is *positive* in this sector.

It is worth emphasizing again that a reduction in distortions is not enough to increase TFP. This reduction must come from sectors which are highly distorted initially.

3.4 Sectoral Distortions and Aggregate TFP in the Data

In the first step of our empirical analysis we take the two distortions in the use of inputs described before as primitives and use the establishment level data from the EIA to measure them. The data is annual and aggregated to the 4 digit NAICS classification. Once we exclude sectors with missing information, we have a total of 82 sectors within Manufacturing. Each of these sectors is mapped into a sector in the model (so $n = 82$).

The result is a panel of sectoral distortions and other variables for the 2003-10 period, whose statistical properties we report. Using this information, we perform the accounting exercise following the decomposition in equation (3) to assess the quantitative contribution of sectoral distortions to aggregate TFP changes. The results show that the components associated to distortions account for a large fraction of the variation of aggregate TFP over time.

3.4.1 Measuring Distortions

For each sector, we have data on gross output, employment, the wage bill, intermediate goods purchased, investment and depreciation. Except for employment, all these variables are measured at current prices. When necessary, we use an aggregate PPI for manufacturing (to deflate gross output, capital and investment) and an aggregate index for the price of intermediate goods to deflate the purchase of intermediates.[10] To construct our measure of the capital stock we use the perpetual inventory method. We use initial investment and a steady-state assumption to calculate the initial capital stock. We then update the capital stock using investment flows and a sector specific depreciation rate.

The construction of our measure of distortions follows the formulas in (5). Notice that since factor shares are not independent of distortions, we cannot identify the production function coefficients with these shares. Our strategy is to take the factor shares from the corresponding sectors i the U.S., as an example of an undistorted economy. For each sector i and at each period t, distortions are then computed as:

$$\theta_{(K,L),t}^i = \frac{\alpha^{i,us}}{1 - \alpha^{i,us}} \left(\frac{nominal \ wage \ bill \ _t^i}{0.14 \times nominal \ capital \ stock \ _t^i} \right)$$

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[10] Ideally, we would like to use sector specific deflators for output and capital goods deflators for capital stock, but the lack of sufficiently disaggregated price indices prevents us from doing so.
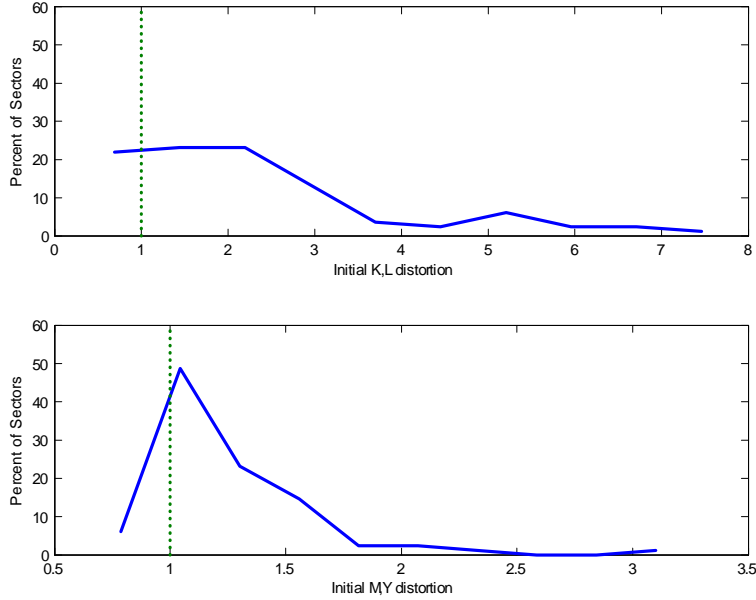
Figure 4: Initial Distribution of Distortions (2003)

assuming a 14 percent rate of return (obtained of the sum of the average real interest rate on loans and the average depreciation rate) and

$$\theta^i_{(M,Y),t} = \left(1 - \varepsilon^{i,us}\right) \left(\frac{nominal\ gross\ output\ {}^i_t}{nominal\ intermediates\ purchases\ {}^i_t} \right).$$

The production function (1) allows us to compute the sectoral technology level A^i_t as

$$A^i_t = \frac{Y^i_t}{\left[\left(K^i_t\right)^{\alpha^{i,us}} \left(L^i_t\right)^{1-\alpha^{i,us}} \right]^{\varepsilon^{i,us}} \left(M^i_t\right)^{1-\varepsilon^{i,us}}}$$

where all variables (except employment) are deflated as described above. Aggregation of sectors is performed as in the model.

3.4.2 Sectoral Distributions

Figure 4 plots the histogram for each type of distortions in the initial year. The intermediates distortions exhibit a large mass of sectors around the level of one (no distortion) and a fat right tail, indicating a majority of sectors with distortions that *reduce* intermediates to output ratios. The capital-labor distortion is more uniformly distributed, with a large majority of sectors (about 80 percent) with distortions that reduce the capital to labor ratios.

Figures 5 and 6 plot the histograms for the *changes* in the two types of distortions, in the subperiods 2003-08 and 2009-10. As observed, most sectors reduced their levels of distortions in the period previous to the crisis, although there is substantial heterogeneity across sectors. In the crisis period the distribution of

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Figure 5: Distribution of Changes in Capital-Labor Distortions

Figure 6: Distribution of Changes in Intermediates Distortions

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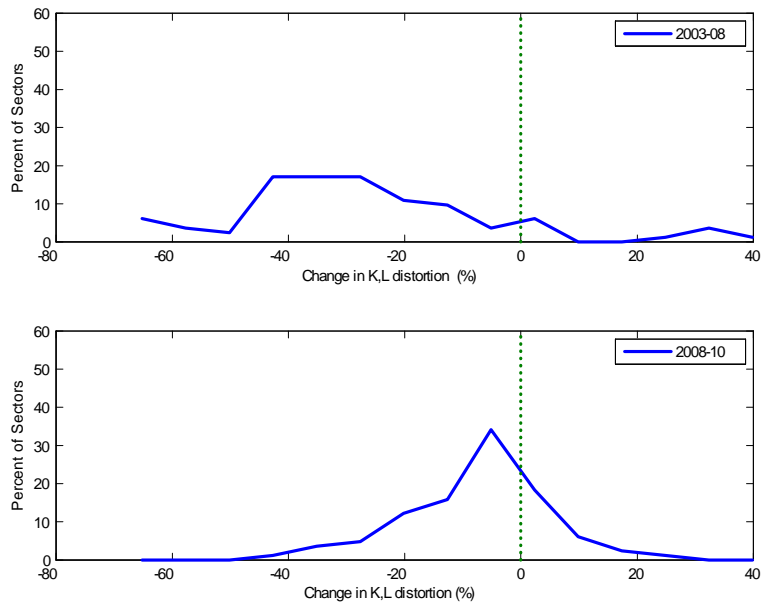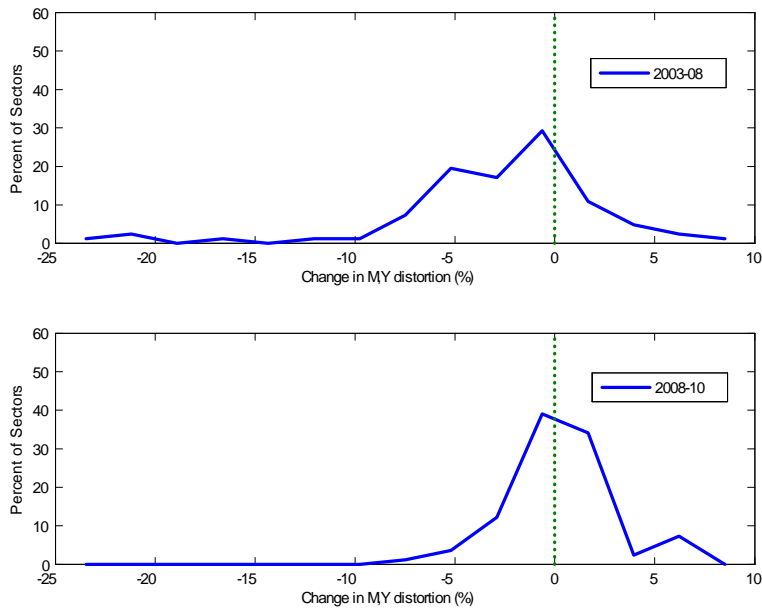|	2003	2005	2008	2009	2010
Average $\theta_{K,L}$	2.36	2.21	1.68	1.49	1.54
Std/Mean $\theta_{K,L}$	0.69	0.73	0.77	0.75	0.74
Correl $(\theta_{K,L},L)$	0.02	-0.03	0.00	0.05	0.02
Correl $(\theta_{K,L},A)$	0.19	0.15	0.16	0.21	0.20
Average $\theta_{M,Y}$	1.25	1.22	1.21	1.21	1.21
Std/Mean $\theta_{M,Y}$	0.30	0.30	0.30	0.30	0.31
Correl $(\theta_{M,Y},L)$	0.15	0.17	0.17	0.18	0.15
Correl $(\theta_{M,Y},A)$	0.20	0.19	0.33	0.37	0.36
Correl $(\theta_{K,L},\theta_{M,Y})$	0.12	0.18	0.20	0.26	0.21

Table 1: Descriptive Statistics for Sectoral Distortions in Mexico's Manufacturing

(yearly % changes)	2003-05	2005-08	2008-09	2009-10
TFP growth	1.90	1.22	-7.32	0.73
due to technology	0.53	0.72	-10.6	0.04
(a) due to reallocation	-0.08	0.37	1.05	-0.54
(b) due to $\theta_{K,L}$	0.18	0.42	-0.51	1.23
(c) due to $\theta_{M,Y}$	1.00	0.37	0.12	0.66
(a) + (b) + (c)	1.10	1.16	0.67	1.35
residual	0.27	-0.65	2.64	-0.65

Table 2: TFP Growth Decomposition

changes shifts to the right, towards more sectors increasing their level of distortions or reducing them at a slower pace.

An alternative characterization of the distributions of distortions is provided as a set of summary statistics in Table 1. The data shows a decrease in the average level of distortions but an increase in their dispersion across sectors, more marked for $\theta_{K,L}$ than for $\theta_{M,Y}$. Both distortions seem to be correlated with sectoral productivity, suggesting that more productive firms are constrained in their use of inputs, and this correlation seems to be increasing over time.

3.4.3   Distortions and Aggregate TFP

Table 2 reports the results of the decomposition of aggregate TFP growth for several periods. The residual is computed as the actual TFP growth minus the predicted TFP growth due to the four factors mentioned before, according to equations (2) and (3). Omitted factors such as changes in relative prices and factor shares, as well as errors in the approximation, are included in this residual.[11]

As expected, sectoral technologies account for a large fraction of aggregate TFP growth. Excluding the residual, about one-third of the growth in 2003-2005, and 38% in 2005-2008 comes from growth in sectoral technologies. However, changes in the two distortions contribute more than 70% of the growth in TFP in

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[11] As a check the robustness of the results, we repeated the exercise from the previous subsection using a more disaggregated sample from Mexican manufacturing (EIA), at the 6-digit NAICS level. This sample allows us to increase the number of sectors from 82 to 215. The results of the TFP growth decomposition are almost identical.

2003-2005 and 40% in 2005-2008. In other words the reduction of distortions and, to a smaller extent the reallocation of labor, contribute very significantly to the productivity gains in manufacturing in this period.

The crisis of 2008-2009 is reflected in the sharp drop in aggregate TFP. Sectoral technologies themselves fell by even more than the aggregate, however, the fall in aggregate TFP would have been much larger had it not been mitigated by the reallocation of labor. The last column of the table shows the contributions of the sectoral distortions to the recovery. Improvements in $\theta_{K,L}$ and $\theta_{M,Y}$ accounted for all the productivity gains in this period, while improvements in sectoral technologies were negligible. Interestingly, labor reallocation functioned as a drag on the recovery process.

4 Financial Frictions and Sectoral Distortions

We now consider an extension of the simple partial equilibrium model in Section 3, but with financial frictions instead of exogenous distortions. As before, financial frictions introduce wedges between factor prices and marginal products, that can be mapped in our two previous sector specific distortions to the capital to labor ratio and to the ratio of intermediate goods to output. This new framework establishes then a link between credit conditions and distortions. We present some exploratory panel regressions to test for this link in our dataset.

4.1 A Simple Model with Working Capital and Borrowing Constraints

As before, the representative firm in sector i purchases capital and labor services and intermediate goods to produce (gross) output, according to the production function specified in (1). We now introduce two types of financial frictions to firms' optimization problem: a simple working capital constraint and a borrowing limit. Firms have to finance their working capital (including all capital services and intermediates purchases) either through loans from the financial system or loans from suppliers (*trade credit*). Both types of loans are repaid at the end of the period including a sector specific, within period, interest rate ι_t^i that we take as exogenous. In addition, loans from suppliers include an exogenous interest rate premium ρ, reflecting the higher cost of trade credit. On the other hand, loans from the financial system are constrained to be no larger than a fraction ξ_t^i of the value of the firm's sales.

A large literature on trade credit informs our modelling choices. Two empirical facts about this form of financing are well established. First, trade credit is used more intensively by firms who are credit constrained, both in the US[12] and in other countries.[13] It also seems that the use of trade credit is more prevalent in countries with poor enforcement and less developed financial systems. Demirguc-Kunt and Maksimovic (2001) find that the magnitude of bank credit relative to trade credit is higher in countries with more efficient legal systems while Fisman and Love (2003) find that industries with higher dependence on trade credit finance grow faster in countries with weaker financial institutions.

Second, trade credit tends to be substantially more expensive than bank credit. A study of more than 30,000 trade credit transactions by Klapper et. al. (2012) shows that the median annual interest rate on these transactions is 54%. Cotler (2013) documents the importance of trade credit for Mexico using a survey

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[12] See for example, Petersen and Rajan (1994) and (1997).

[13] Gama et. al (2010) find this to be true for a panel of Portugese and Spanish firms, while Couppey-Soubeyran and Hericourt (2011) find that firms in the MENA region that have difficulty in gaining access to bank credit use trade credit instead. Atanasova and Wilson (2003) find that the use of trade credit increases during periods of monetary contractions in the UK.

carried out by the Central Bank of Mexico. He finds that 82% of all businesses report using trade credit to finance their expenses, and most of them use simultaneously bank credit as working capital and trade credit. Our calculations using the same survey (ENAFIN, for its acronym in Spanish) reveals that the cost of this credit was about 5% per month or an annualized rate of close to 80%.

In each period, the problem of the representative firm in each sector i is to solve

$$\max_{L^i_t, K^i_t, M^i_t, \kappa^i_t} p^i_t Y^i_t - w_t L^i_t - \left[1 + \iota^i_t + (1 - \kappa^i_t)\rho\right]\left(r_t K^i_t + p^M_t M^i_t\right) \tag{4}$$

$$s.to. \quad Y^i_t = A^i_t \left[\left(K^i_t\right)^{\alpha^i} \left(L^i_t\right)^{1-\alpha^i}\right]^{\varepsilon^i} \left(M^i_t\right)^{1-\varepsilon^i}$$

$$\kappa^i_t \left(r_t K^i_t + p^M_t M^i_t\right) \leq \frac{\xi^i_t p^i_t Y^i_t}{1 + \iota^i_t}$$

$$0 \leq \kappa^i_t \leq 1,$$

where κ^i_t is the endogenous fraction of working capital financed through the financial system (we call it simply credit from now onwards). Together with the interest rate ι^i_t, the sequence of parameters ξ^i_t captures credit conditions by governing the tightness of the borrowing constraint. These credit conditions affect sectors differently and can change over time. For now, the sequences for factor prices (w_t, r_t, p^M_t) and sectoral output prices (p^i_t) are all exogenous.

4.2 Mapping Credit Conditions into Sectoral Distortions

Since the problem for each representative firm is static, from now on we omit the subscript t. The first order conditions for profit maximization imply

$$\frac{\alpha^i}{1 - \alpha^i}\left(\frac{wL^i}{rK^i}\right) = \left[1 + \iota^i + (1 - \kappa^i)\rho\right] + \kappa^i \lambda^i$$

and

$$(1 - \varepsilon)\left(\frac{p^i Y^i}{p^M M^i}\right) = \left(1 + \iota^i\right) \frac{\left[1 + \iota^i + (1 - \kappa^i)\rho\right] + \kappa^i \lambda^i}{1 + \iota^i + \lambda^i \xi^i}$$

where λ^i is the Lagrange multiplier associated to the borrowing constraint.

From these first order conditions, we can map the sectoral distortions introduced in Section 3 to the new model as

$$\theta^i_{K,L} = \left[1 + \iota^i + (1 - \kappa^i)\rho\right] + \kappa^i \lambda^i$$

$$\theta^i_{M,Y} = \left(1 + \iota^i\right) \frac{\left[1 + \iota^i + (1 - \kappa^i)\rho\right] + \kappa^i \lambda^i}{1 + \iota^i + \lambda^i \xi^i}. \tag{5}$$

Notice that distortions now are endogenous and depend on the sector-specific credit conditions (ι^i, ξ^i) together with two endogenous variables (λ^i and κ^i). These distortions arise because the shadow cost of credit increases the effective cost of capital relative to labor, and of intermediates relative to output, distorting the optimal mix of inputs. In a world without financial frictions these two distortions will disappear.

In Appendix A.4 we further characterize the solution to the firm's optimization problem and show how, depending on the tightness of the borrowing constraint (ξ^i) relative to other parameters, one of the following three cases can arise:

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1. For *high tightness* the borrowing constraint binds but $\kappa^i < 1$. Available bank credit is not sufficient to purchase all inputs and the firm has to resort to the more expensive trade credit. Then,

$$\theta^i_{K,L} = 1 + \iota^i + \rho \qquad \theta^i_{M,Y} = \left(1 + \iota^i\right) \frac{1 + \iota^i + \rho}{1 + \iota^i + \rho \xi^i}.$$

2. For *intermediate tightness* the borrowing constraint binds and $\kappa^i = 1$. Available bank credit allows firms to avoid trade credit, but still their purchase of inputs is constrained. In this case,

$$\theta^i_{K,L} = \left(1 + \iota^i\right) \frac{\left(1 - \xi^i\right)\left(1 - \varepsilon^i\left(1 - \alpha^i\right)\right)}{\xi^i \varepsilon^i \left(1 - \alpha^i\right)} \qquad \theta^i_{M,Y} = \left(1 + \iota^i\right) \frac{1 - \varepsilon^i\left(1 - \alpha^i\right)}{\xi^i}.$$

3. For *low tightness* the borrowing constraint is not binding and $\kappa^i = 1$. There is no constraint to the firms' purchase of inputs and all of it is financed thorugh bank credit. Therefore,

$$\theta^i_{K,L} = \theta^i_{M,Y} = 1 + \iota^i.$$

4.3 Credit Conditions and Sectoral Distortions in the Data

In this model, it is easy to show that an increase in the sector specific interest rate always increases both types of distortions. Also, an increase in the borrowing limit decreases the two distortions until the point in which the firm becomes unconstrained. We now test these predictions in our dataset, matching bank credit data aggregated to the 4 digit sector level to measured distortions at the same level of aggregation. As a measure of credit, we use the short term flow of credit to the sector inclusive of interest liabilities.[14] We measure real interest rates as the interest rate on the median loan in the sector in the current year, deflated by the change in the manufacturing price level.[15]

4.3.1 Distortions in the Intermediates to Output Ratio

Table 3 shows the results for the distortions on intermediate goods. Each row of the table represents a series of regressions, each with the independent variable in the left hand side column. The first three columns show a simple OLS with time dummies, while the next three show fixed effects regressions. Columns (7) to (9) show fixed effects augmented with time effects and the last three columns show regressions of each variable interacted with time dummies. For brevity, only the interactions for the 2008-2010 period are shown. Heteroscedasticity consistent standard errors are given below the estimates.

The first panel shows regressions with the credit to output ratios, where the denominator is obtained from the sectoral data of the EIA, while the next two panels are the regressions with measures of credit flow and interest rates respectively. In each case we also present estimates with additional controls for sector size or productivity.

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[14]We also use an alternative measure of short term credit net of interest liabilities with almost identical results.

[15]We also used an alternative measure of interest rate which is the average interest rate, weighted by the size of the loan. However, this measure is likely to be biased downwards since larger loans are associated with lower interest rates in the data.

Table 3: The Intermediate Goods Distortion

Dependent Variable $\theta^i_{M,Y}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A:												
Credit/Output	-0.1024**	-0.0928*	-0.0584	-0.001	0.0003	-0.0005	0.0023	0.0029	0.0021			
	0.0529	0.0512	0.0535	0.0105	0.0104	0.0105	0.0102	0.0101	0.0102			
Credit/Output ×2008										-0.0145	-0.0144	-0.0149
										0.0132	0.0131	0.0132
×2009										-0.0024	-0.0001	-0.0013
										0.013	0.0128	0.0130
×2010										-0.0101	-0.008	-0.0095
										0.0146	0.0144	0.0146
Labor			2.6995†			1.3611**			0.6482			0.9951
			0.6798			0.7328			0.7377			0.7285
A		0.3808†			0.0697†			0.0618†			0.0657†	
		0.0564			0.0181			0.0175			0.0177	
Panel B:												
Credit Flow	0.0116†	0.0088†	0.0075**	-0.0012*	-0.0012*	-0.0013*	-0.0011*	-0.0011*	-0.0011*			
	0.0032	0.0031	0.0034	0.0008	0.0007	0.0008	0.0007	0.0007	0.0007			
Credit Flow ×2008										-0.0018**	-0.0018**	-0.0019**
										0.0008	0.0008	0.0008
×2009										-0.0011	-0.001	-0.0009
										0.0012	0.0012	0.0012
×2010										-0.0029**	-0.0027**	-0.0028**
										0.0014	0.0014	0.0014
Labor			2.2574†			1.4457**			0.7575			1.0900*
			0.7161			0.7324			0.7389			0.7449
A		0.3674†			0.0699†			0.062†			0.0649†	
		0.0561			0.0181			0.0175			0.0177	
Panel C:												
Interest Rate	0.038†	0.0332†	0.0442†	-0.0018†	-0.0019†	-0.0019†	0.0002	0.0005	0.0003			
	0.0091	0.0088	0.009	0.0005	0.0005	0.0005	0.0013	0.0013	0.0013			
Interest Rate ×2008										0.0001	0.0003	0.0001
										0.0014	0.0014	0.0013
×2009										0.0005	0.001	0.0007
										0.0016	0.0016	0.0016
×2010										0.0002	0.0007	0.0004
										0.0017	0.0017	0.0017
Labor			3.3166†			1.6411†			0.6537			0.8007
			0.0561			0.7285			0.7380			0.7333
A		0.3674†			0.074†			0.0621†			0.0603†	
		0.0561			0.018			0.0175			0.0176	
Sector Effects	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Dummies	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes

Note: $*p < 0.15$, $**p < 0.05$, $†p < 0.01$ Heteroscedasticity consistent standard errors below.

Columns (1) to (3) of Panel A shows that credit intensity and $\theta_{M.Y}$ are negatively related. This seems to suggest that the use of credit is an important source of minimizing input distortions. Concerns about the endogeneity of credit intensity could arise if more productive sectors, or sectors with larger collateral have smaller distortions to input use and also have more access to credit. However, as columns (2) and (3) show, our results are robust to the inclusion of additional controls such as sector size (measured by number of employees) or sectoral productivity. Interestingly, more productive sectors also have larger distortions, suggesting that the removal of these distortions would have large effects on output.

Columns (4) to (6) and (10) to (12) show that the sign of the estimates is not altered if we include sectoral heterogeneity and time varying coefficients. In all cases the coefficient is not significant, but given that the numerator and the denominator of the credit to output ratio come from different sources, we expect a certain degree of variability, reflected in the large standard errors.

Panel B studies the effects of actual short term credit flow in the period. While the coefficient is positive when we don't include sectoral effects (Columns (1) to (3)), it is negative once sectoral heterogeneity is taken into account. In the last three columns, where we consider time varying coefficients, we find that the availability of credit matters for the size of the distortions in both the crisis and the recovery years. Sectors which were able to secure credit were able to bring their intermediate goods usage closer to the optimal level.[16] In all cases, results are robust to additional controls of productivity and labor.

Finally, panel C considers the cost of the credit, as measured by the median interest rate on short term credit in the sector. The first three columns show that this is positively and significantly related to the distortion, suggesting that higher interest rates prevent firms from using their optimal input mix. The estimates remain positive when we introduce sectoral heterogeneity (Columns (7) through (9)) but they are no longer significant. The last three columns show that while the coefficient was positive even in the crisis and recovery years, it is not significant. One concern here of course is how to create a representative interest rate for a group of loans in a sector. We have experimented with alternate interest rate measures such as averages of the interest rates on all short term loans in the period, weighted by loan size, but this measure has the drawback that it may over-represent low interest loans and be biased downwards. We also plan to experiment with averages weighted by loan maturity in further research.

4.3.2   Distortions in the Capital Labor Ratio

Table 4 shows the relationship between the capital labor distortion and indicators of credit intensity, availability and cost. The table is organized in a similar fashion as Table 3. In general, panel A shows that sectors with a greater credit intensity have lower distortions. In particular, the last three columns of this panel indicate that greater credit intensity in the crisis and recovery implied that firms could get closer to their undistorted optimal capital labor ratio. As in the previous estimations, more productive sectors and sectors with more employees have bigger distortions in their capital labor ratio.

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[16]Column (10)-(12) in all tables is estimated with a full set of interactions (2003-2010) but only the last three years are shown for compactness. None of the interactions in the previous years are significant. Time varying coefficients were estimated for all other independent variables but yielded no results of interest. All results available on request from authors.

Table 4: The Capital Labor Distortion

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						Dependent Variable $\theta^i_{K,L}$						
Panel A:												
Credit/Output	-0.3536	-0.3198	-0.3739	-0.4665**	-0.4243†	-0.4290**	-0.0886	-0.0747	-0.1024			
	0.2899	0.2863	0.2965	0.2024	0.1958	0.1939	0.1625	0.1588	0.1606			
Credit/Output ×2008										-0.368*	-0.3657*	-0.3699*
										0.2382	0.2307	0.2311
×2009										-0.7091†	-0.6416†	-0.6224†
										0.2335	0.2264	0.2269
×2010										-0.5487**	-0.4881**	-0.5047**
										0.2623	0.2542	0.2545
Labor			-1.2466			98.8465†			1.0507			77.031†
			3.7696			13.5808			1.2608			12.7292
A		1.333†			2.2033†			1.4612†			1.9348†	
		0.3154			0.3429			0.2754			0.313	
Panel B:												
Credit Flow	-0.0789†	-0.0909†	-0.0914†	-0.0108	-0.0114	-0.0179	0.0002	-0.0003	-0.0038			
	0.0173	0.0171	0.0186	0.0148	0.0143	0.0142	0.0117	0.0114	0.0116			
Credit Flow ×2008										-0.0233*	-0.0237	-0.0297**
										0.0159	0.0154	0.0159
×2009										-0.0667†	-0.0613†	-0.0475**
										0.0228	0.0221	0.0223
×2010										-0.0868†	-0.0804†	-0.0810†
										0.0275	0.0267	0.0266
Labor			6.9892*			100.816†			45.1968†			86.7127†
			3.9149			13.6467			11.6517			13.879
A		1.5729†			2.2302†			1.4634†			2.0507†	
		0.3119			0.344			0.2754			0.3326	
Panel C:												
Interest Rate	0.1885†	0.1721†	0.1922†	0.0111	0.0069	0.0032	0.0259	0.0333*	0.0307*			
	0.0494	0.049	0.0499	0.0106	0.0103	0.0102	0.0207	0.0202	0.0204			
Interest Rate ×2008										-0.0002	0.0042	0.0037
										0.022	0.0216	0.0217
×2009										-0.0519**	-0.0389*	-0.0358**
										0.0253	0.0249	0.0252
×2010										-0.0339	-0.0225	-0.0221
										0.0272	0.0268	0.0269
Labor			1.9893†			98.6277†			46.1721†			50.6616†
			3.6686			13.5349			11.4459			11.5424
A		1.274†			2.2015†			1.4962†			1.3923†	
		0.3119			0.3404			0.2715			0.2777	
Sector Effects	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Dummies	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes

Note: $^*p < 0.15$, $^{**}p < 0.05$, $^†p < 0.01$. Heteroscedasticity consistent standard errors below.

The relationship between distortions and credit flow is presented in panel B. The negative relation between these two indicates that financial constraints are an important factor underlying distortions. As columns (10) to (12) show, the availability of credit was particularly important in the recovery from the crisis. In other words, sectors with greater availability of credit were able to reduce their input distortions during the crisis. Given the contribution of the distortions to aggregate TFP, as evidenced in table 2 this result points to the relationship between financial frictions and TFP, through the effect of the former on input allocation.

Finally the lowest panel shows the effect of interest rates on the capital labor distortion. An increase in the cost of borrowing is associated with an increase in distortions and is significant in several cases. However, the cost of credit as measured by the median interest rate in the sector does not seem to play an important role in the recovery, suggesting that it was the availability of credit, rather than its cost, which was important in reducing distortions. As mentioned earlier, it is probably worth experimenting with alternative measures of the cost of credit to establish the robustness of these results. It may also be that credit rationing occurs by quantity, not price, in the market for loans and so the cost of credit is not as important as its availability.

4.3.3 In Summary

The two sets of estimates in tables 3 and 4 taken together, highlight the role of credit in explaining the distortions on input use. Both the flow of credit and the credit intensity are negatively related to the size of distortions in the optimal use of intermediates and the capital labor ratio. As mentioned earlier, we have already seen the importance of these distortions in explaining aggregate TFP. While that was an informative accounting exercise, the estimates presented in this section give some content to what lies behind the distortions. Our results suggest that the amount of credit available to the sector is an important determinant of its ability to achieve its best input mix.

We also find that credit plays an especially important role in the recovery of the economy from the 2008 crisis. This is noteworthy because this recovery takes place without a corresponding increase in aggregate real credit and aggregate credit intensity, as shown by Figure 2. Our results therefore shed light on an important puzzle in economics, i.e. the phenomenon of "creditless recoveries". As Calvo et. al. (2006) document, output in many emerging economies recovered after financial crises without a corresponding recovery in credit. However, our estimates show that recoveries can take place as long as credit goes to sectors that can reduce their distortions, regardless of the aggregate level of credit.

5   Financial Frictions and Aggregate TFP

The previous two sections have shown an indirect connection between credit conditions and aggregate TFP, via the effect of financial frictions on sectoral distortions and inputs use. In this final section we put the pieces together and analyze the impact of exogenous credit shocks calibrated from the data on aggregate TFP. In particular, we ask the question of how much can we account of the movements of TFP in the data using these credit shocks as primitives, instead of the sectoral distortions (as in Section 3). For this, we embed the model of firms' optimization with working capital and borrowing constraints from Section 4 in a general equilibrium

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structure.[17] The model is calibrated using our dataset and used to perform counterfactual experiments.

5.1 The General Equilibrium Model

We consider an economy with n sectors producing manufacturing goods under perfect competition. Each of these producers faces the sequence of static optimization problems described in equation (4) above. The output of each sector is used either for the production of the composite final good or the composite intermediate good ($Y_t^i = Y_t^{i,F} + Y_t^{i,M}$). These composite goods are also produced under perfect competition using the constant returns to scale aggregators

$$Y_t = \left(\sum_{i=1}^{n} \omega^i (Y_t^{i,F})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

for the final good, with elasticity of substitution $\eta > 1$ and $\sum_{i=1}^{n} \omega^i = 1$, and

$$M_t = \left(\sum_{i=1}^{n} \omega^i (Y_t^{i,M})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

for the intermediate good. From standard first order conditions we construct the price of the final good

$$p_t = \left(\sum_{i=1}^{n} \left(\frac{1}{\omega^i} \right)^{-\eta} (p_t^i)^{1-\eta} \right)^{\frac{1}{1-\eta}} = 1$$

from the prices of the intermediate goods produced by each sector.[18] The price of the final good is the numeraire.

A representative consumer makes all savings and investment decisions in a small open economy setup. The consumer maximizes the standard intertemporal utility

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the budget constraint

$$C_t + I_t + B_{t+1} - B_t = \sum_{i=1}^{n} \left(w_t L_t^i + r_t K_t^i \right) + r_t^* B_t + T_t.$$

The consumer receives income from renting capital and labor to the manufacturing sectors. An additional source of income comes from the interest payments on trade credit, which are rebated as a lump sum transfer

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[17] We need a general equilibrium model of the economy beacuse, as shown in Section 3, the reallocation of factors across sectors is key for this accounting exercise. With constant returns to scale, the relative size of sectors is determined from the demand side. Also, changes in factor prices would affect the relation between sectoral shocks and aggregate TFP.

[18] From the same set of first order conditions we obtain the demand for each sector i's output in the final good production

$$Y_t^{i,F} = \left(\frac{1}{\omega^i} \right)^{-\eta} (p_t^i)^{-\eta} Y_t$$

and in the intermediate good production

$$Y_t^{i,M} = \left(\frac{1}{\omega^i} \right)^{-\eta} (p_t^i)^{-\eta} \phi_t M_t.$$

Under constant returns to scale, these demand functions allow us to pin down the size of each sector i and its change over time.

to the consumer:

$$T_t = \sum_{i=1}^{n} (1 - \kappa_t^i)(\iota_t^i + \rho)(p_t^M M_t^i + r_t K_t^i)$$

The consumer's income is used for consumption, investment and international borrowing/lending. Capital follows the standard law of motion

$$K_{t+1} = (1 - \delta)K_t + I_t.$$

The consumer has access to an asset B_t that allows her to borrow and lend internationally at the exogenous interest rate r_{t+1}^*. Notice that capital accumulation and international borrowing and lending are the only dynamic decisions in this model.

The model is closed by the market clearing conditions for the final good, which can be used for consumption, investment and net exports

$$Y_t = C_t + I_t + NX_t,$$

for the intermediate good, used as an input by the manufacturing sectors,

$$M_t = \sum_{i=1}^{n} M_t^i,$$

and for capital and labor

$$K_t = \sum_{i=1}^{n} K_t^i \qquad L_t = \sum_{i=1}^{n} L_t^i.$$

The balance of payments identity in he model equates net exports plus interest payments to the change in assets. Interest payments include those made by producers in each sector i, as we assume that domestic financial intermediaries obtain resources from abroad:

$$NX_t + r_t^* B_t - \sum_{i=1}^{n} \kappa_t^i \iota_t^i (r_t K_t^i + p_t^M M_t^i) = B_{t+1} - B_t.$$

5.2   Calibration

From the analysis in Section 3, we already have factor shares α^i, ε^i for each of the 82 sectors (taken as their U.S. counterparts) and a panel of sectoral technologies A_t^i, computed as the sectoral Solow residuals in each year of the sample. We calibrate the shares ω^i as the proportion of each sector's sales in the total value of manufacturing output in 2003. Notice that while we allow sectoral productivities to change within sectors and over time, we keep factor shares and output shares constant over time. To finish with the production side, we set $\eta = 1$, i.e., an unitary elasticity of substitution across manufacturing goods.

The panel for interest rates ι_t^i, is the sector specific real interest rate which will represent one of our exogenous parameters affecting credit conditions. The other parameter related to credit is the tightness of the borrowing limit ξ_t^i. To identify this parameter we need to assume that the borrowing constraint is binding for each sector and in each year.[19] Then, we can construct

$$\xi_t^i = (1 + \iota_t^i) \left(\frac{\text{short term credit}_t^i}{\text{gross output}_t^i} \right)$$

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[19] As we mentioned in the previous section, in our model the borrowing constraint would be binding in two out of three cases. Theoretically, this depends on the value of ξ_t^i being strictly smaller than $\Psi^i = 1 - \varepsilon^i + \alpha^i \varepsilon^i$. We check whether this is true or not. If it is not, then we set $\xi_t^i = \Psi^i$. In this way we eliminate the third case and assume that the borrowing constraint is always binding or tangent. In the data we find that the third case, the borrowing constraint not binding, occurs only for a small fraction (less than 10 percent) of sectors and years.

(Yearly TFP growth %)	2003-05	2005-08	2008-09	2009-10
Data	1.92	1.24	-7.32	0.73
Baseline experiment	1.71	1.12	-12.10	1.47
Model with changes only in:				
(a) Technology (A_t^i)	1.37	0.04	-10.25	0.82
(b) Technology and borrowing limit	1.28	0.14	-10.22	0.42
(c) Technology and interest rate	1.80	1.03	-12.15	1.88
(d) Technology and *average* ξ_t and ι_t	0.92	-0.48	-10.79	0.16

Table 5: TFP Growth Predictions of the Baseline Model

combining the credit data from R04 and the real data from EIA.

An important parameter in the model is the interest rate premium ρ for loans from suppliers which we do not have in our data. As mentioned earlier, estimates of the cost of trade credit in Mexico from the ENAFIN suggest a median annual rate of almost 80%, mostly in the form of early payment discounts. Petersen and Rajan (1994) document that a substantial fraction of firms do not take advantage of these discounts. In our calibration, we will therefore use a conservative 15% value of ρ and test for sensitivity using a range of other values.

For the remaining parameters, we chose a real international interest rate of 7 percent and an annual depreciation rate of 7 percent (implying from the usual arbitrage conditions a rental price for capital of 14 percent). These values roughly correspond to the average interest rate and depreciation rates across sectors and over time in our dataset, and they are consistent with the computation of the distortion to the capital to labor ratio in Section 3. In a steady state, the interest rate pins down the (inverse of) the discount factor. We normalize the inelastic supply of labor to one and assume a steady state stock of foreign assets equal to zero.

5.3 The Baseline Experiment

The baseline experiment is obtained by feeding the model with the exogenous panels for technologies, borrowing limits and interest rates obtained from the data as explained in the calibration description. We solve the model as a sequence of steady states, one for each year of the sample. As highlighted before, this is a model of a small open economy in which the only dynamic decision is with respect to the aggregate capital stock. Therefore, looking only at steady states instead of a transitional dynamics would not have a significant impact in the results about TFP and sectoral distortions.

5.3.1 Aggregate TFP

Table 5 reports the results of the baseline model with respect to changes in aggregate TFP over time, computed as in Table 2. The experiment feeding the model with the three exogenous panels (A_t^i, ξ_t^i and ι_t^i) is reported in the second row and compared to the data, reported in the previous row. The baseline model performs well in explaining the evolution of TFP in 2003-2008, accounting for almost 90% of observed TFP growth in this period. The model overpredicts both the fall of TFP in 2009 and the subsequent recovery in the following year.

What were the contributions of each of these exogenous shocks? To answer this question, we conduct a series of counterfactual experiments, the results of which are reported in the rows labelled (a), (b), (c) and (d). In experiment (a) we solve the model keeping the credit related parameters equal to their 2003 levels and allow only the technology parameter A_t^i to vary over time. Experiment (b) consists of varying A_t^i and ξ_t^i as in the data and keeping ι_t^i at its 2003 levels. This gives us an idea of the role of the borrowing limits. Experiment (c) explores the role of the variation in the cost of credit ι_t^i by keeping the borrowing limit ξ_t^i at its 2003 levels. Finally experiment (d) explores the role of reallocation of credit across sectors. We eliminate sector-specific variation in interest rates and borrowing limits by fixing them at their average values each period for all sectors.[20]

Experiment (a) shows that in the growth period of 2003-2008 the contribution of technology varies from about 80% in the first sub period to almost nothing in the second. In this period, as experiment (c) suggests, adding the variation in interest rates as observed in the data increases the growth of TFP substantially. This suggests that sector-specific interest rates play an important role in TFP movements, a result also found by Pratap and Urrutia (2012) at the aggregate level. The role of the borrowing limit is mixed. Experiment (b) shows that it was not very important in the 2003-2005 period. However increases in ξ_t^i , and the corresponding increase in credit to output ratios in 2005-2008, did contribute to TFP growth.

Let us now consider the periods of crisis and recovery. As experiments (a) and (c) show, the largest contributors to the movements of TFP are the variation in the sectoral technologies and the interest rates. Variations in the credit to output ratio (as captured by the parameter ξ_t^i) function as a drag to the growth of TFP.

Finally, experiment (d) shows the role of sectoral heterogeneity in explaining TFP movements. As the results reveal, reallocation of credit and interest rates across sectors play a large role. In its absence, TFP growth would have been about half of the model predicted growth in 2003-2005, and negative in 2005-2008. In the recovery period, without reallocation, our model predicts that TFP growth would have been a mere 0.16%, as opposed to 1.47% with it.

Taken together, our results point to a considerable role for financial factors in explaining the movements in aggregate TFP. The allocation of credit and interest rates across sectors is particularly important. During the recovery period of 2010, substantial reallocation occurs against a backdrop of declining total credit. Our results suggest however, that a fall in credit is not necessarily detrimental to the economy if the existing credit is reallocated optimally.

5.3.2   Additional Results

Table 6 assess the performance of the model in other dimensions. Our model is able to match the mean values of both distortions quite well, although the model predicted distortions are substantially less than what is observed in the data. We also see that the model predicted credit and the credit intensity are negatively related to the distortions, with correlations very similar to those observed in the data. The correlations between real variables such as output and labor and the distortions are also very much in line with what is observed in the data

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[20]Specifically, we calculate the average interest rate and the average borrowing limit across sectors for each year, and their growth rate throughout 2003-2010. Then we take the initial 2003 values for these two variables in the data for each sector and use the growth rate to construct a panel. Therefore we are keeping the distribution across sectors constant over time.

	$\theta^{it}_{M,Y}$		$\theta^{it}_{K,L}$	
	Data	Model	Data	Model
Mean	1.22	1.14	1.94	1.16
Std. Deviation	0.37	0.05	1.47	0.05
Correlation with:				
Y_{it}	-0.02	-0.03	-0.11	-0.06
L_{it}	0.16	0.12	0.003	0.019
$(\text{Credit})_{it}$	-0.13	-0.26	-0.19	-0.23
$\left(\frac{\text{Credit}}{\text{Output}}\right)_{it}$	-0.08	-0.15	-0.06	-0.06

Table 6: Comparison of Baseline Model with Data

(Yearly TFP growth %)	2003-05	2005-08	2008-09	2009-10
Data	1.92	1.24	-7.32	0.73
Alternative experiment with $\rho = 0.05$	1.92	0.63	-11.29	1.38
Model with changes only in:				
(a) Technology (A^i_t)	1.37	0.04	-10.25	0.82
(b) Technology and borrowing limit	1.36	0.06	-10.23	0.73
(c) Technology and interest rate	1.94	0.61	-11.31	1.46
(d) Technology and *average* ξ_t and ι_t	1.01	-0.40	-10.73	0.23
Alternative experiment with $\rho = 0.25$	1.52	1.44	-12.51	1.23
Model with changes only in:				
(a) Technology (A^i_t)	1.37	0.04	-10.25	0.82
(b) Technology and borrowing limit	1.18	0.24	-10.19	-0.04
(c) Technology and interest rate	1.72	1.25	-12.58	2.10
(d) Technology and *average* ξ_t and ι_t	0.86	-0.53	-10.84	0.11

Table 7: Sensitivity Analysis: Trade Credit Premium

5.4 Sensitivity Analysis

Finally, we report the results of sensitivity analysis on the trade credit premium ρ. As mentioned when presenting the calibration, we used a value of 0.15 in the baseline experiment. In the first counterfactual we lower the premium to 5%. In the second alternative experiment we increase it to 25%. The results are reported in Table 7. The main message of these exercises is that our main results are robust to reasonable changes in this parameter.

The top panel of Table 7 shows results for $\rho = 0.05$. The change in predicted TFP in the first subperiod increases compared to the value in Table 5, but in the rest of the subperiods the predicted change is smaller compared to what was previously found. The variation in results is due to the interaction between interest rates, borrowing constraint parameters, and the smaller premium on trade credit. This interaction produces different values for the distortions on input use in each subperiod. The same message comes out from the second panel of Table 7, for $\rho = 0.25$, although now the model accounts for slightly less of the change in predicted TFP during 2003-2005 and more in the remaining subperiods. The magnitudes are in both cases

comparable to the results for the baseline experiment. Experiment (c) and (d) continue to show how changes in interest rates over time, and across sectors, play an important quantitative role accounting fo the behavior of TFP.

6   Conclusions

Several studies have analyzed the role of firm-specific distortions in the use of capital, labor and intermediates in accounting for differences in total factor productivity across countries. We focus instead on the impact of changes in these type of distortions in the evolution of TFP over time. First, using data for Mexican manufacturing industries and a theory-based TFP decomposition, we show that distortions account for a large fraction of aggregate TFP changes between 2003 and 2010. Second, merging the manufacturing survey with data on bank loans, we show that a strong empirical relationship exists between changes in distortions and changes in the availability and the cost of credit. Third, we construct a general equilibrium model to measure the contribution of changes in financial frictions on changes in TFP. The model accounts for a large fraction of observed TFP growth between 2003-2008, although it overpredicts both the fall of TFP in 2009 and the subsequent recovery in the following year. Taken together, the results suggest an important connection between credit conditions and aggregate productivity channeled through the choice of the inputs mix by firms.

It is worth highlighting that our analysis is conducted at the sectoral level, not at the firm level. Our unit of analysis is a narrowly defined sector within manufacturing, modelled as a representative firm operating a constant returns to scale technology. Hence, in contrast with most of the literature on idiosyncratic distortions and TFP, we abstract from differences in distortions among firms within the same sector. This is arguably a limitation of our analysis driven by the data availability. However, it also helps us to focus on the sectoral margin and isolate the impact of distortions on the optimal input mix from issues related with the optimal size of firms. Our results show the quantitative importance of this margin.

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A Appendix

A.1 Crosswalk between INEGI AND CNBV data

The sector of economic activity in the loan level data from December 2001 to June 2009 is classified according to an internal CNBV classification.The data for the period July 2009 to July 2012, like that of the EIA, is classified according to the more standard NAICS 2007. To map the earlier R04 data into the NAICS 2007 classification we need a crosswalk that tells us how to reclassify each category.

The credit data we have was provided by the CNBV. We did not receive the disaggregated data which contains each particular credit issued during the December 2001-July 2012 period but were given the disaggregated (and anonymized) data for the period January 2009-December 2009. This data is especially useful for our purpose since it contains individual credit data for 6 months before and after the classification system changed. We used this data to build the crosswalk using a *revealed reclassification* method in which we make the mapping among both classifications by observing where each credit was originally classified and were it was reclassified once the classification system changed between June and July 2009.

We build a crosswalk by observing the reclassifications that actually took place in the data. The reason for building the crosswalk in this way instead of in a more arbitrary manner is that here we can take into account what actually happened and, in some sense, try to extract the crosswalk that was used when the reclassification was made and which is not available to us.

There are 1066 categories in the R04 data while there are 598 categories at the 5-digit level in the NAICS 2007 data. This means that in order to do a complete mapping, several R04C categories might be mapped into the same NAICS 2007 category. An additional problem is that the crosswalk we observe from the data is not deterministic in the sense that each credit in the R04C is not always reclassified to the same NAICS 2007 category, rather the credits in each R04C category are reclassified into a small subset of NAICS 2007 categories (and to some more often than to others). Given this second problem we built a probabilistic crosswalk which lets us know into which categories we have to reclassify each R04C category and also tells us exactly how much we have to put into each.

As a brief illustrative example suppose we want to know how to reclassify the data from category 100000 in the R04C to the NAICS 2007. Suppose that in the disaggregated data we have 10 different credits classified to category 100000 between January and June 2009. Next suppose we see that once the reclassification takes place, we observe that 5 of these credits were reclassified during July and December 2009 into NAICS 2007 category 11111, 4 were reclassified to category 11112 and only 1 was reclassified into 11113. Then, the crosswalk would tell us that the data in category 100000 of the R04C should be distributed among categories 11111, 11112 and 11113 of the NAICS 2007 and the weights should be 50%, 40% and 10% respectively.

As mentioned previously the R04C data has 1066 categories, but when building the probabilistic crosswalk we were only able to map 995 of them. The remaining 71 categories were not mapped in this way because there were no credit observations in the disaggregated data that were originally classified into these categories and then reclassified to another in the NAICS 2007 (this happens if we have no credit observations for one of the 71 categories at all or if we only have them for the period January-June 2009). Fortunately we were able to use the catalog of the R04C to match 32 of the missing 71 categories into the NAICS 2007. To do this we matched them to the category whose name seemed more appropriate. For these 32 categories the

crosswalk is deterministic as they were assigned fully to a single NAICS 2007 category. The remaining 39 categories in the R04 were not matched because they are missing in the catalog and thus cannot be mapped in this way either.

A.2 EIA Data

Data definitions for the real variables are given below:

Gross Output is defined as the value of all production. This was cross-checked against an alternative value of gross output, namely the value of sales of the establishment plus change in inventories of finished goods.

Intermediate Goods are defined as the sum of expenditures on raw materials, packaging, fuels and energy.

Capital Stock is constructed using the perpetual inventory method. We use initial investment and a steady-state assumption to calculate the initial capital stock. We then update the capital stock using investment flows and a sector specific depreciation rate.

Labor is the sum of all male and female personnel employed directly and indirectly by the establishment. The latter includes labor provided by independent contractors.

Value Added is computed as gross output less intermediate goods. The former is deflated using the manufacturing PPI and the latter using the intermediate goods deflator.

A.3 Obtaining TFP Growth Accounting Expression (3) in Section 3

We can rewrite firms' first order conditions as:

$$\frac{K^i}{L^i} = \left(\frac{\alpha^i}{1-\alpha^i}\right)\left(\frac{w}{r}\right)\frac{1}{\theta_{K,L}^i} \qquad \frac{M^i}{Y^i} = \left(1-\varepsilon^i\right)\left(\frac{p^i}{p^M}\right)\frac{1}{\theta_{M,Y}^i}$$

so that

$$Y^i = \left(A^i\right)^{\frac{1}{\varepsilon^i}}\left[\left(\frac{\alpha^i}{1-\alpha^i}\right)\left(\frac{w}{r}\right)\frac{1}{\theta_{K,L}^i}\right]^{\alpha^i}\left[\left(1-\varepsilon^i\right)\left(\frac{p^i}{p^M}\right)\frac{1}{\theta_{M,Y}^i}\right]^{\frac{1-\varepsilon^i}{\varepsilon^i}}L^i.$$

In growth rates

$$\widetilde{K^i} = \widetilde{L^i} - \widetilde{\theta_{K,L}^i} \qquad \widetilde{M^i} = \widetilde{Y^i} - \widetilde{\theta_{M,Y}^i} \tag{1}$$

where $\widetilde{x}_t = \log\left(\frac{x_t}{x_0}\right) \approx \frac{x_t - x_0}{x_0}$. This implies

$$\widetilde{Y^i} = \left(\frac{1}{\varepsilon^i}\right)\widetilde{A^i} - \alpha^i\widetilde{\theta_{K,L}^i} - \left(\frac{1-\varepsilon^i}{\varepsilon^i}\right)\widetilde{\theta_{M,Y}^i} + \widetilde{L^i} \tag{2}$$

assuming that factor shares *and* relative prices remain constant over time. This expression allows us to decompose changes in sectoral output into changes in technology, changes in the two distortions and changes in employment. From the definition of aggregate TFP in the text, we can write

$$\widetilde{TFP} = \left(\frac{Y_0}{Y_0 - p_0^M M_0}\right)\widetilde{Y} - \left(\frac{p_0^M M_0}{Y_0 - p_0^M M_0}\right)\widetilde{M} - \alpha\widetilde{K} - (1-\alpha)\widetilde{L} \tag{3}$$

assuming again that factor shares and relative prices remain constant over time. Aggregation implies

$$\widetilde{Y} = \frac{1}{Y_0}\sum_{i=1}^n \left(p_0^i Y_0^i\right)\widetilde{Y^i}, \qquad \widetilde{M} = \frac{1}{M_0}\sum_{i=1}^n M_0^i\widetilde{M^i}$$

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and

$$\widetilde{K} = \frac{1}{K_0} \sum_{i=1}^{n} K_0^i \widetilde{K^i}, \qquad \widetilde{L} = \frac{1}{L_0} \sum_{i=1}^{n} L_0^i \widetilde{L^i}.$$

Replacing in (3), aggregate TFP growth can be written in terms of sectoral variables as

$$\widetilde{TFP} = \sum_{i=1}^{n} \left[\left(\frac{p_0^i Y_0^i}{Y_0 - p_0^M M_0} \right) \widetilde{Y^i} - \left(\frac{p_0^M M_0^i}{Y_0 - p_0^M M_0} \right) \widetilde{M^i} - \alpha \left(\frac{K_0^i}{K_0} \right) \widetilde{K^i} - (1-\alpha) \left(\frac{L_0^i}{L_0} \right) \widetilde{L^i} \right]. \qquad (4)$$

Finally, replace (1) and (2) in (4) to obtain

$$\widetilde{TFP} = \sum_{i=1}^{n} \left\{ \omega^i \left(\frac{1}{\varepsilon^i} \right) \left(\widetilde{A^i} \right) + \left[\omega^i - \alpha \left(\frac{K_0^i}{K_0} \right) - (1-\alpha) \left(\frac{L_0^i}{L_0} \right) \right] \widetilde{L^i} \right.$$
$$\left. + \left[\alpha \left(\frac{K_0^i}{K_0} \right) - \omega^i \alpha^i \right] \widetilde{\theta_{K,L}^i} + \left[\frac{p_0^M M_0^i}{Y_0 - p_0^M M_0} - \omega^i \left(\frac{1-\varepsilon^i}{\varepsilon^i} \right) \right] \widetilde{\theta_{M,Y}^i} \right\}$$

with $\omega^i \equiv \frac{p_0^i Y_0^i - p_0^M M_0^i}{Y_0 - p_0^M M_0}$ representing the share of value added of sector i in the aggregate.

A.4   Characterizing the Solution to the Firm's Problem (4) in Section 4

Omitting the time subscript and sector superscript, the Lagrangian for this problem is

$$pY - wL - (1 + \iota + (1-\kappa)\rho)\left(rK + p^M M\right) + \lambda \left[\frac{\xi pY}{1+\iota} - \kappa \left(rK + p^M M \right) \right] + \mu(1-\kappa)$$

with

$$Y = A \left[(K)^\alpha (L)^{1-\alpha} \right]^\varepsilon (M)^{1-\varepsilon}.$$

where λ is the multiplier of the borrowing constraint and μ the multiplier of the restriction $\kappa \leq 1$. The first order conditions are

$$L \;:\; (1-\alpha)\varepsilon p \frac{Y}{L} - w + \lambda (1-\alpha)\varepsilon p \frac{Y}{L} \left(\frac{\xi}{1+\iota} \right) = 0$$

$$K \;:\; \alpha\varepsilon p \frac{Y}{K} - (1 + \iota + (1-\kappa)\rho) r + \lambda \left[\alpha\varepsilon p \frac{Y}{K} \left(\frac{\xi}{1+\iota} \right) - \kappa r \right] = 0$$

$$M \;:\; (1-\varepsilon) p \frac{Y}{M} - (1 + \iota + (1-\kappa)\rho) p^M + \lambda \left[(1-\varepsilon) p \frac{Y}{M} \left(\frac{\xi}{1+\iota} \right) - \kappa p^M \right] = 0$$

$$\kappa \;:\; \rho \left(rK + p^M M \right) - \lambda (rK + p^M M) - \mu = 0$$

with complementary slackness conditions:

$$\lambda \left(\frac{\xi pY}{1+\iota} - \kappa r K^i + \kappa p^M M \right) \;=\; 0, \qquad \text{with } \lambda \geq 0$$
$$\mu(1-\kappa) \;=\; 0, \qquad \text{with } \mu \geq 0.$$

Notice that with constant returns to scale the first three first order conditions are equivalent to the production function, so the level of output Y is indetermined (obtained from the demand side of the economy). Simplifying the FOC, we obtain the system

$$(1-\alpha)\varepsilon p \frac{Y}{L}\left(1+\frac{\lambda\xi}{1+\iota}\right) = w \tag{5}$$

$$\alpha\varepsilon p \frac{Y}{K}\left(1+\frac{\lambda\xi}{1+\iota}\right) = r\left[(1+\iota+(1-\kappa)\rho)+\kappa\lambda\right] \tag{6}$$

$$(1-\varepsilon) p \frac{Y}{M}\left(1+\frac{\lambda\xi}{1+\iota}\right) = p^M\left[(1+\iota+(1-\kappa)\rho)+\kappa\lambda\right] \tag{7}$$

$$(rK+p^M M)(\rho-\lambda) = \mu \tag{8}$$

$$\lambda\left[\frac{\xi pY}{1+\iota}-\kappa\left(rK+p^M M\right)\right] = 0 \tag{9}$$

$$\mu(1-\kappa) = 0. \tag{10}$$

with unknowns K, L, M, κ, λ and μ, and given prices w, r, p^M, ι, ρ and output Y. To solve for the multipliers, replace (6) and (7) in (9) to obtain

$$\lambda\left\{\frac{\xi pY}{1+\iota}-\kappa\left[\frac{1+\iota+\lambda\xi}{(1+\iota)\left[(1+\iota+(1-\kappa)\rho)+\kappa\lambda\right]}\right](1-\varepsilon+\alpha\varepsilon)\,pY\right\}=0$$

or

$$\lambda\left\{\xi-\kappa\left[\frac{1+\iota+\lambda\xi}{(1+\iota+(1-\kappa)\rho)+\kappa\lambda}\right](1-\varepsilon+\alpha\varepsilon)\right\}=0$$

which means that either $\lambda=0$ or

$$\kappa\Psi\left[\frac{1+\iota+\lambda\xi}{(1+\iota+(1-\kappa)\rho)+\kappa\lambda}\right]=\xi$$

with $\Psi\equiv 1-\varepsilon+\alpha\varepsilon$, $0<\Psi<1$.Therefore,

$$\lambda=\max\left\{\frac{\kappa(1+\iota)(\Psi-\xi)-(1-\kappa)\xi(1+\iota+\rho)}{\kappa\xi(1-\Psi)},0\right\} \tag{11}$$

Similarly, replacing (6) and (7) in (8)

$$\mu=\max\left\{\left[\frac{1+\iota+\lambda\xi}{(1+\iota)\left[(1+\iota+(1-\kappa)\rho)+\kappa\lambda\right]}\right]\Psi pY(\rho-\lambda),0\right\} \tag{12}$$

These last two equations will allow as to characterize the different cases or corner solutions.

Case 1: $\lambda>0,\mu=0$ In the first case the borrowing constraint is binding, but $\kappa\leq 1$. Using (8), we obtain $\lambda=\rho$. Subsitituting in the other FOC, we obtain

$$(1-\alpha)\varepsilon p \frac{Y}{L}\left(1+\frac{\rho\xi}{1+\iota}\right) = w$$

$$\alpha\varepsilon p \frac{Y}{K}\left(1+\frac{\rho\xi}{1+\iota}\right) = r(1+\iota+\rho)$$

$$(1-\varepsilon)p \frac{Y}{M}\left(1+\frac{\rho\xi}{1+\iota}\right) = p^M(1+\iota+\rho)$$

$$\kappa\left(rK+p^M M\right) = \frac{\xi pY}{1+\iota}$$

with unknowns K, L, M and κ. Notice that, from (11), it has to be the case that

$$\frac{\kappa(1+\iota)(\Psi-\xi)-(1-\kappa)\xi(1+\iota+\rho)}{\kappa\xi(1-\Psi)}=\rho$$

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therefore
$$\kappa = \frac{(1 + \iota + \rho)\,\xi}{(1 + \iota + \rho\xi)\,\Psi}$$

This can only be a solution if $\kappa \leq 1$, this is, if
$$\xi \leq \frac{\Psi\,(1 + \iota)}{1 + \iota + (1 - \Psi)\,\rho}$$

depending on parameter values.

Case 2: $\lambda > 0, \mu > 0$ In this case both constraints are binding. As $\kappa = 1$, the system of FOC becomes

$$
\begin{aligned}
(1 - \alpha)\varepsilon p\frac{Y}{L}\left(1 + \frac{\lambda\xi}{1 + \iota}\right) &= w \\
\alpha\varepsilon p\frac{Y}{K}\left(1 + \frac{\lambda\xi}{1 + \iota}\right) &= r(1 + \iota + \lambda) \\
(1 - \varepsilon)p\frac{Y}{M}\left(1 + \frac{\lambda\xi}{1 + \iota}\right) &= p^M(1 + \iota + \lambda) \\
(rK + p^M M)(\rho - \lambda) &= \mu \\
rK + p^M M &= \frac{\xi pY}{1 + \iota}
\end{aligned}
$$

with unknowns K, L, M, λ and μ. From (11) and (12), we must have
$$\lambda = \frac{(1 + \iota)\,(\Psi - \xi)}{\xi\,(1 - \Psi)}$$

and
$$\mu = \left[\frac{1 + \iota + \lambda\xi}{(1 + \iota)\,(1 + \iota + \lambda)}\right]\Psi pY\,(\rho - \lambda)$$

To have $\lambda > 0$ and $\mu > 0$ simultaneously we then need
$$0 < \frac{(1 + \iota)\,(\Psi - \xi)}{\xi\,(1 - \Psi)} < \rho$$

or
$$\frac{\Psi(1 + \iota)}{(1 + \iota) + (1 - \Psi)\rho} < \xi < \Psi$$

again depending on parameter values.

Case 3: $\lambda = 0, \mu > 0$ In this case $\kappa = 1$ but the borrowing constraint is not binding. The system of FOC becomes

$$
\begin{aligned}
(1 - \alpha)\varepsilon p\frac{Y}{L} &= w \\
\alpha\varepsilon p\frac{Y}{K} &= r(1 + \iota) \\
(1 - \varepsilon)p\frac{Y}{M} &= p^M(1 + \iota) \\
(rK + p^M M)\rho &= \mu
\end{aligned}
$$

with unknowns K, L, M and μ. From (12), we must have

$$\mu = \left[\frac{1 + \iota + \lambda \xi}{(1 + \iota)(1 + \iota + \lambda)} \right] \Psi p Y \rho > 0$$

which is always satisfied. On the other hand, from (11), we must have

$$\frac{(1 + \iota)(\Psi - \xi)}{\xi(1 - \Psi)} \leq 0$$

or $\Psi \leq \xi$, depending on parameter values.

Case 4: $\lambda = 0, \mu = 0$ Using equation (8), this would imply

$$rK + p^M M = 0,$$

but that requires at least one negative price or quantity. Therefore, we rule it out.

In Summary We have three relevant cases depending on parameters, in particular on how tight the borrowing constraints is:

- For high tightness $\xi \leq \frac{\Psi(1+\iota)}{1+\iota+(1-\Psi)\rho}$, Case 1 $(\lambda > 0, \mu = 0)$ occurs.

- For intermediate tightness $\frac{\Psi(1+\iota)}{1+\iota+(1-\Psi)\rho} < \xi < \Psi$, Case 2 $(\lambda > 0, \mu > 0)$ occurs.

- For low tightness $\xi \geq \Psi$, Case 3 $(\lambda = 0, \mu > 0)$ occurs.

Implications for Distortions Notice that, in general, from FOC (5) - (7) we can write distortions as:

$$\begin{aligned}
\theta_{K,L} &\equiv \frac{\alpha}{1-\alpha} \frac{w}{r} \frac{L}{K} = [1 + \iota + (1-\kappa)\rho] + \kappa\lambda \\
\theta_{M,Y} &\equiv (1-\varepsilon)\frac{p}{p^M}\frac{Y}{M} = (1+\iota)\frac{[1 + \iota + (1-\kappa)\rho] + \kappa\lambda}{1 + \iota + \lambda\xi}.
\end{aligned}$$

depending on κ and the multiplier λ (but not on μ). Then, in the different cases:

- Case 1: Since $\lambda = \rho$, we obtain

$$\begin{aligned}
\theta_{K,L} &= 1 + \iota + \rho \\
\theta_{M,Y} &= (1+\iota)\frac{1+\iota+\rho}{1+\iota+\rho\xi}.
\end{aligned}$$

- Case 2: Since $\kappa = 1$ and $\lambda = \frac{(1+\iota)(\Psi-\xi)}{\xi(1-\Psi)}$, we obtain

$$\begin{aligned}
\theta_{K,L} &= 1 + \iota + \frac{(1+\iota)(1-\xi)\Psi}{\xi(1-\Psi)} = (1+\iota)\frac{(1-\xi)\Psi}{\xi(1-\Psi)} \\
\theta_{M,Y} &= (1+\iota)\frac{1+\iota+\frac{(1+\iota)(1-\xi)\Psi}{\xi(1-\Psi)}}{1+\iota+\frac{(1+\iota)(\Psi-\xi)}{(1-\Psi)}} = (1+\iota)\frac{\Psi}{\xi}
\end{aligned}$$

- Case 3: Since $\kappa = 1$ and $\lambda = 0$, we obtain

$$\theta_{K,L} = \theta_{M,Y} = 1 + \iota$$